A Theory of Multiplexity: Sustaining Cooperation with Multiple Relations *

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Abstract

We investigate the overlap and formation of multiple social relations. We model these relations as repeated prisoner's dilemmas with varying stakes. Agents decide whether to connect with a friend or a stranger and set the cooperation level for each relationship. We define "multiplexity" as the choice to link with a friend. We find a strong tendency towards multiplexity, leading to a potential "multiplexity trap" where agents prefer linking with friends over more efficient strangers; additionally, multiplexity tends to prevail in networks with low degree dispersion or positive assortativity. Empirical evidence from the Indian Village Survey supports our theoretical predictions.

Keywords: Multiplex, Networks, Cooperation, Network formation, Informal economy

JEL Classification Numbers: C73, D85, O17, Z13

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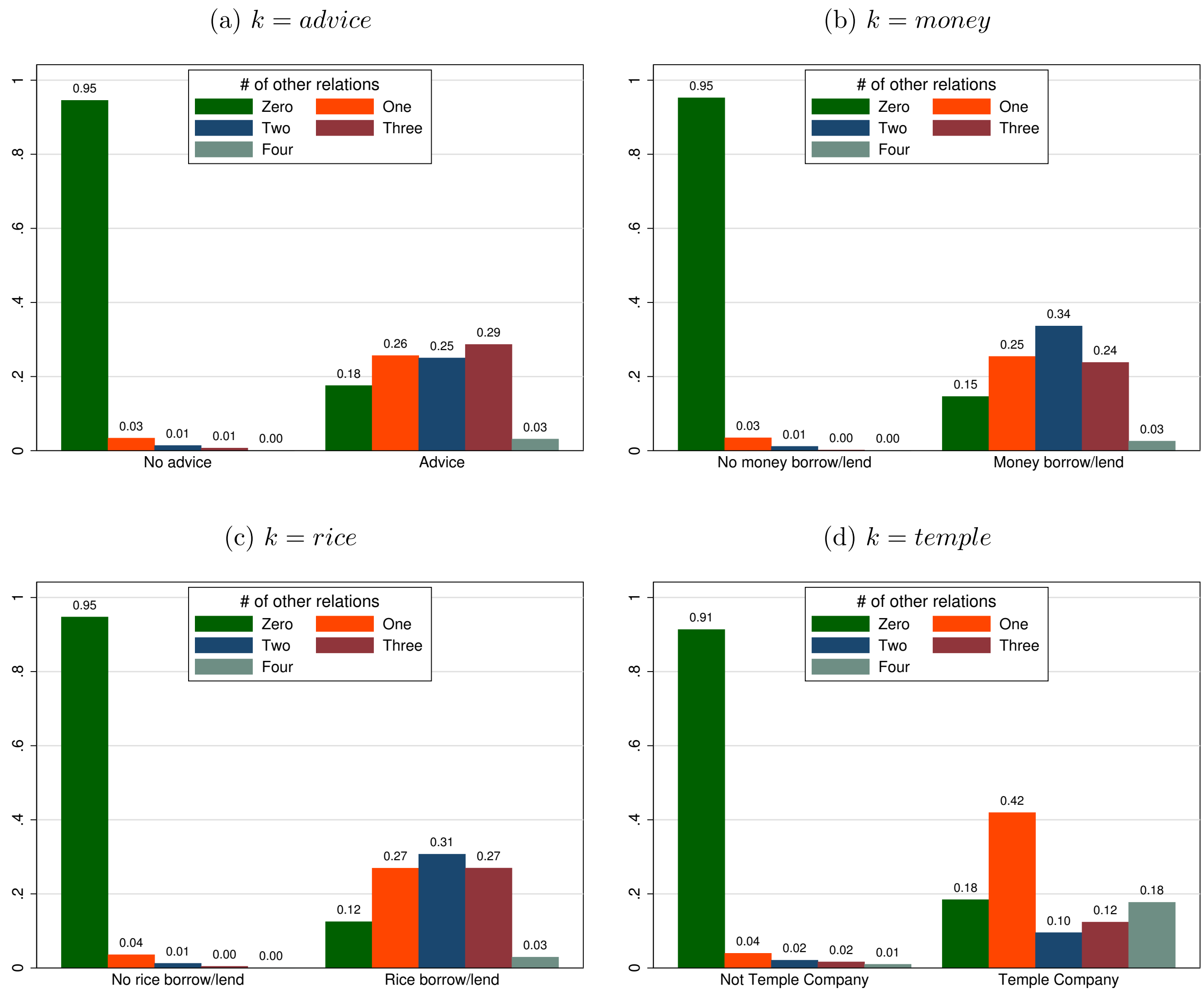
1 Introduction

People are intricately connected through various social interactions and relationships. These connections manifest in numerous ways, such as shared meals at restaurants, collaborative research endeavors, financial transactions like borrowing and lending money, and more. For example, in a typical northern Chinese village, the complex social fabric can include up to 22 distinct connections that shape the lives of its inhabitants (Yan, 1996). Similarly, extensive surveys in Indian villages (Banerjee et al., 2013) have revealed a multitude of relationships, from temple visits to kerosene exchanges.

Despite the prevalence of multiple relationships, there is a dearth of systematic studies examining their interconnections.[1] Observationally, some societies show a tendency for people to interact more with existing friends, indicating overlapping relationships, while others do not. This paper explores the concept of multiplexity in relationships both theoretically and empirically. We first demonstrate (in Figure 1) that multiplexity, or the overlapping of different relations, is common in the 75 villages surveyed in the Indian Village Survey Data (Banerjee et al., 2013; Jackson, Rodriguez-Barraquer and Tan, 2012). We then develop a theoretical model to endogenize the formation of multiplexity in relationships, focusing on network features influencing the choice between a friend and a stranger. Empirical data from the surveys provide supportive evidence for our theoretical predictions.

[1] As McPherson, Smith-Lovin and Cook (2001) note, despite the seminal study by Fischer (1982) on networks in North California, there remains a scarcity of large-scale research on overlapping networks of different relationship types. A recent survey by Atkisson et al. (2020) echoes this sentiment, highlighting the limited work on multilayer and multiplex networks.

Figure 1: Prevalence of Multiplexity



Note: Each panel compares the histograms of the number of other relationships, conditional on having a given relationship k (left) or not (right). The panels share the same pattern: when a pair of households do not have the given relationship, it is highly likely (above 90%) that they do not have any other relationships, whereas when they have the given relationship, it is highly likely (above 80%) that they also have at least one of the other relationships.

Model Setup. We model relationships as repeated prisoner's dilemmas with variable stakes, in line with Ghosh and Ray (1996) and Ali and Miller (2016).[2] In each relationship, cooperation opportunities arise sequentially over time with a Poisson arrival rate λ, occurring independently across different relationships and pairs. For instance, in a coauthorship relationship, opportunities like manuscript revisions, conference deadlines, and grant appli-

[2]These models, with their varying stakes of cooperation, facilitate measuring the effectiveness of different networks in sustaining cooperation and enable clearer comparisons across networks.

cations arise randomly over time. In each stage of the game, agents first choose the stakes of cooperation, such as the quality of a coauthored paper, and then decide to cooperate or defect. Payoffs can vary across pairs, and different relationships can have varying importance.

Among all subgame perfect equilibria, we focus on those that achieve the maximal stakes of cooperation (MSC), where agents simultaneously attain their highest payoffs. We demonstrate the existence of such equilibria and present an algorithm for constructing them in any given network. This focus on MSC equilibria simplifies the comparison of different networks' effectiveness in fostering cooperation.

We examine a network formation process where, upon the rare emergence of a new relationship, an agent chooses with whom to establish it, either a friend (to multiplex) or a stranger.[3] Following the link choice, agents determine the stakes of cooperation in each relationship based on the network structure, aiming to maximize their equilibrium payoff in the resultant network.

Tradeoff. A key tradeoff faced by agents when choosing between a friend and a stranger is between multiplexity and community enforcement. Linking with a current friend enhances the stakes of cooperation across all relationships, as multiple connections serve as social collateral. In contrast, linking with a stranger may better utilize community enforcement, which depends more on the broader network structure.[4]

Results. We observe a natural inclination towards linking with friends, which can lead to a 'multiplexity trap' where agents continually add relations with current friends, even when linking with strangers would be more efficient. We show that if societies begin with isolated pairs and the compatibility of cooperation across different pairs is relatively uniform, agents will persist in linking with friends, resulting in societies remaining as isolated pairs indefinitely.

We further explore network structures, identifying degree dispersion and assortativity as key determinants of the choice between a friend and a stranger. Specifically, individuals tend to multiplex either when the network exhibits low degree dispersion (similar number of friends for all individuals) or high degree dispersion coupled with positive assortativity (agents linked with those having a similar number of friends). In these scenarios, the incentives to multiplex dominate.

Conversely, in networks with negative assortativity, agents with many connections may prefer adding links with strangers who also have a high degree, as this can significantly

[3]This assumption ensures that agents can disregard future link dynamics. The implications of relaxing this assumption are discussed in Section 5.

[4]Canonical models of community enforcement include those by Kandori (1992) and Ellison (1994). Also, numerous other factors influence this choice, including homophily, access to new information, congestion costs, and more. We acknowledge these complexities and discuss our specific angle and contributions at the end of the introduction and in the literature review.

enlarge the cooperation scope. Asymmetry in degree, both within the broader society and among neighbors, is crucial for agents to break free from the multiplexity trap and consider linking with strangers.

Our theory extends in various directions, most notably by allowing for the importance of different relationships to vary, thereby exploring the impact of asymmetric relationships on multiplexity. We find that people are more likely to link with friends when the new relationship is of greater importance, as existing relationships can support and be supported by the new one.[5] This finding contributes to the ongoing debate on trust versus access (see, for example, Granovetter, 1973 and Karlan et al., 2009), suggesting that for less important new relationships (e.g., those primarily for information access), agents may be more inclined to connect with strangers.

Summary and Testable Hypotheses. In summary, our theoretical analysis endogenizes the formation of multiple social relations, emphasizing the tradeoff between multiplexity and community enforcement. We generate new predictions about how existing network patterns influence overlapping relationships. Our hypotheses, derived from our theoretical results, include:

- Multiplexity is prevalent in networks (Hypothesis 1).
- Multiplexity is more likely in networks with:
    - ⋄ Low degree dispersion (Hypothesis 2a).
    - ⋄ Positive assortativity (Hypothesis 2b).

In the final part of the paper, we test our theoretical predictions using the Indian Village Survey data by Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). This dataset provides detailed network information on multiple types of relationships for each of the 75 surveyed Indian rural villages. Our empirical analysis strongly supports the existence of multiplexity, consistent with our predictions regarding how network patterns affect the choice of multiplexity, namely, its negative association with degree dispersion and positive association with assortativity.

We also conduct a robustness check using caste information from the dataset, given the significant role of the caste system in Indian culture (Hsu, 1963).[6] We hypothesize that cooperation among villagers belonging to the same subcastes is primarily influenced by factors like religion, which our model does not account for. Conversely, the incentives proposed in our model are more pertinent for villagers from different subcastes. The empirical results

[5] This approach also addresses the distinction between multiplexity and the intensification of a single link, as it allows for an exploration of the asymmetry in different relationships.

[6] We are grateful to Kaivan Munshi for suggesting this robustness check.

are consistent with our hypothesis. To distinguish our theory from alternative explanations such as homophily, we accounted for similarities among agents in other dimensions, including education level and occupation type. Our theoretical predictions remain significant even after introducing these controls.

Discussion. The embeddedness of human interactions is a vast topic, and this paper can only address a small aspect of it. We do not aim to provide a comprehensive analysis of why people sometimes prefer linking with a friend rather than a stranger. The reasons are multifaceted, including homophily, information access, insurance benefits, uncertainty reduction, congestion cost avoidance, third-party enforcement strength, and more.

Our focus is on a simplified model that abstracts many complex real-world factors to concentrate on how network features might influence the choice between a friend and a stranger. This perspective provides a new lens through which to view multiplexity patterns and distinguishes our theory from previous ones.

The rest of the paper is organized as follows: In Section 1.1, we review the literature. Section 2 sets up the baseline model, focusing on the incentives to cooperate within the network structure. We analyze the incentive effects of multiplexity in Section 2.2 and compare it with community enforcement in Section 2.3. Section 3 presents a simple model of network formation, exploring our main question: 'when to multiplex, and when not to.' We demonstrate the possibility of a multiplexity trap in Section 3.1. Section 4 explores how network features might affect the multiplexity choice. In Section 5, we extend our theory in various directions. Section 6 is dedicated to conducting the empirical analysis. Finally, Section 7 concludes the paper.

1.1 Literature Review

Anthropologists and sociologists have long underscored the significance of multilayer or multiplex networks. Kaplan et al. (1985) contend that the absence of reciprocal food sharing implies a broader scope of reciprocity. Uzzi (1997) notes that long-term partnerships, or embedded ties, often stem from prior personal relationships, integrating economic exchanges with friendship and altruism.

In economics, Bernheim and Whinston (1990) analyze multimarket contexts, finding that diverse markets enhance firms' cooperative behaviors. Li and Powell (2020) reveal that non-deterministic environments, even with identical multimarket scenarios, can bolster cooperation. Chen, Zenou and Zhou (2018) delve into social network games where agents engage in dual activities, highlighting the impact of individual productivity and network density on outcomes. Kor and Zhou (2023) investigate optimal interventions in networks with multiple activities. These works typically assume the multiplexity of relationships, whereas our study focuses on its endogenous formation.

Recent research on network structuring and multiplexity has been gaining traction.[7] On network effects on behavior, Tzavellas (2023) studies shock propagation in multi-layer networks. On network formation, Joshi, Mahmud and Sarangi (2020) explore the formation of a new layer based on inherited structures, observing a 'silver spoon' effect where initial centrality influences future network status. Billand et al. (2021) examine the formation of two-layer networks, analyzing stability in relation to the characteristics of payoffs, and Banerjee et al. (2023) find that the introduction of micro-finance disrupts not only borrowing-lending links but also various other, less directly relevant, social relationships. Our research diverges by emphasizing how multiplexity jointly determines agents' behavior and network formation:[8] having multiple types of relationships boosts agents' incentives for cooperation, and this, in turn, provides strong incentives for multiplexity. This framework allows us to compare multiplexity with community enforcement in providing incentives, and to identify key network features such as degree dispersion and assortativity that determine the tendency towards multiplexity.

Additionally, our study contributes to the extensive literature on community enforcement and informal insurance in networks (see Wolitzky, 2020 for an overview), finding that the drive towards multiplexity can impede complete network formation. Model-wise, we expand upon Ali and Miller (2016) by incorporating the formation of multiple relations in a complete information setting, focusing on the interplay between multiplexity and community enforcement.

2 Baseline Model with Fixed Networks

Consider a society in which there are n agents represented by $N = \{1, ..., n\}$. Also, there are K relationship-networks, $G = (g^1, \dots, g^K)$, and $K > 1$. All the K networks are undirected and unweighted, i.e., $g^k_{ij} = g^k_{ji} \in \{0, 1\}$, $\forall i, j, k$. In particular, $g^k_{ij} = 1$ means that agents i and j have a link in relationship k, and $g^k_{ij} = 0$ otherwise. We say $ij \in G^k$ iff $g^k_{ij} = 1$.

Let $N^k_i(G) = \{j \neq i \mid ij \in G^k\}$ be the set of neighbors of agent i in relationship k. Let $d^k_i(G) = |N^k_i(G)|$ be i's degree (i.e., number of neighbors) in relationship k. We simplify

[7] There is also an emerging literature on multiplex networks in fields other than economics. Interested readers may refer to Lee, Min and Goh (2015) and Atkisson et al. (2020) for an overview. Unlike earlier works focusing on homophily or constraints in expanding networks, our approach centers on incentive spillovers from multiple relations and the endogenization of multiplexity.

[8] Our work is in dialogue with literature on endogenous partnership formation and subsequent game dynamics. Genicot and Ray (2003), Bloch, Genicot and Ray (2008), and Jackson and Watts (2010) have laid foundational work in this area, while recent studies like Xing (2021) and Sadler and Golub (2022) advance the understanding of stable matchings and combined strategies in network formation and game equilibria. All these papers focus on the formation of single-layer networks.

the notations as N_i^k and d_i^k, respectively, when there is no confusion. An agent i's entire set of neighbors (across all the relationships) is $N_i \equiv \bigcup_k N_i^k$, and this agent's *total degree* is $d_i = \sum_k d_i^k$. We note that d_i is i's total number of links, which may not equal the total number of their neighbors, since an agent can have more than one link with each neighbor under multiplexity. We do not distinguish link and relationship in the paper, and will use the terms interchangeably throughout.

We model each relationship as a repeated prisoner's dilemma with endogenous stakes. This framework follows closely a series of work by Ali and Miller (2013, 2016, 2022). They examine an incomplete information setting to address the information diffusion issue in networks, whereas we focus on perfect monitoring case to investigate the multiplexity choice. The benefits of the framework is that it allows for a transparent comparison across different equilibria with a fixed discount factor. Specifically, an infinitely repeated game is played in each relationship. Cooperation opportunities (i.e., stage games) arrives sequentially over time, following a Poisson process with arrival rate $\lambda > 0$, which is independent across links/relationships. For example, in a co-authoring relationship, cooperation opportunities, such as conference deadline, revision request, etc., arrive randomly over time. When each cooperation opportunity arises, the ij pair plays the following extensive-form stage game:

(i) First, agents simultaneously propose *the stakes of cooperation*, say the intended quality of a paper, $\phi_i, \phi_j \geq 0$; the minimum of the two, $\phi = \min\{\phi_i, \phi_j\}$, is selected.

(ii) Then, they play the following prisoner's dilemma:

	Cooperate	Defect
Cooperate	ϕ, ϕ	$-V(\phi), T(\phi)$
Defect	$T(\phi), -V(\phi)$	$0, 0$

in which $V(\phi) > 0$ and $T(\phi) > \phi, \forall \phi > 0$. We assume $T(0) = V(0) = 0$, $T'(0) = 1$ and $\lim_{\phi\to\infty} T'(\phi) = \infty$. This implies $T(\phi)/\phi$ increases from close to 1 to infinity as ϕ changes from 0 to infinity. In other words, the larger the stakes of cooperation, the higher the deviation payoff. Throughout this paper, we work with the case in which

$$V(\phi) = \phi \text{ and } T(\phi) = \phi + \phi^2$$

to obtain explicit expressions of the key variables. Most insights extend to general forms of $T(\cdot)$ and $V(\cdot)$.

We note several things here. First, stakes could vary across ij-pairs and different relationships k. That is, ϕ can be indexed as ϕ_{ij}^k. We simplify the notation and will omit the subscripts and superscript when there is no ambiguity. Second, we could also multiply a "compatibility index" $c_{ij}^k > 0$ in front of the above payoff matrix. It captures the idea that some pairs can be more compatible than others for certain tasks and/or that different relationships can vary in their importance. The compatibility index has no effect when there is only one link but would shape the incentives when there are multiple relationships. We assume uniform compatibility and normalize c_{ij}^k to 1 for most part of the paper, but will allow c_{ij}^k to vary with ij and k when we discuss efficient network formation and the implications of varying relationship importance.

All agents discount the future with a common factor r. Therefore, the discounted payoff of future cooperation with stake ϕ between two agents is as follows:

$$\phi \int_0^\infty e^{-rt} \lambda \mathbf{d}t = \frac{\phi\lambda}{r}.$$

The society of the K networks and the compatibilities are common knowledge among all agents. Then, the payoff of an agent with k relationships and j friends is as follows:

$$\int_0^\infty e^{-rt} \lambda \mathbf{d}t \sum_{j,k} c_{ij}^k \phi_{ij}^k 1_{\{ij \in G^k\}}.$$

Finally, we assume perfect monitoring across the entire society, so that any deviation is detected immediately by all society.

Discussion of Assumptions. Firstly, it is important to note that with given cooperation stakes, there are constant returns to scale in the number of relationships. Assuming increasing returns would favor multiplexity, whereas decreasing returns would incline agents towards linking with strangers. One possible manifestation of decreasing returns is the presence of congestion costs in multiplexing: as the number of relationships among a pair of friends increases, the energy (time, resources, etc.) dedicated to each specific relationship might decline. This naturally limits the benefits of multiplexing and steers individuals towards forming connections with strangers. On the other hand, it is also realistic to consider the costs associated with meeting or getting to know a stranger. For the sake of focus, we overlook these costs in our analysis, while acknowledging their significance in practical scenarios.

Secondly, our analysis is based on a perfect monitoring environment, where the past actions of everyone are commonly observed by all. This environment has two notable aspects: the perfect monitoring assumption and the public monitoring across the entire society. We will address the perfect monitoring aspect in our discussion of grim-trigger strategies. For now, we focus on public monitoring, which we believe is plausible in the closely-knit networks under examination (refer to Breza and Chandrasekhar, 2019; Banerjee et al., 2019). Assuming that past histories are private among interacting friends could weaken community enforcement (as evidenced in studies by Ali and Miller, 2016, 2022), thereby potentially favoring multiplexity, which relies less on information about the broader community. Conversely, this assumption may also imply that players might not fully realize the benefits of completing informational cycles.[9] The direction of outcomes may vary significantly once we deviate from the public monitoring environment, and the final result will depend on how imperfect information specifically affects multiplexing and community enforcement. We find this topic intriguing and complex enough to warrant a separate paper for a comprehensive discussion.

2.1 Equilibrium and Maximal Stakes of Cooperation (MSC)

We consider subgame perfect equilibria and, particularly, those with grim-tigger strategies which provide the strongest incentives to cooperate.[10] Formally:

Definition 1 (Equilibrium with grim-trigger strategies). *Given the society (networks) G, an equilibrium (with grim-trigger strategies) is characterized by the stakes of cooperations, $\{\phi_{ij}^k\}_{g_{ij}^k=1}$, such that on equilibrium path, all agents on all relationships/links always choose to cooperate. Off-path, if any agent ever deviates in any relationship/link, they are punished by all their neighbors who cease cooperating in any link/relationship with them. Specifically, the equilibria are characterized by $\{\phi_{ij}^k\}_{g_{ij}^k=1}$'s which satisfy the following incentive constraints:*

$$c_{ij}^k\phi_{ij}^k + c_{ij}^k(\phi_{ij}^k)^2 \leq c_{ij}^k\phi_{ij}^k + \int_0^\infty e^{-rt}\lambda \boldsymbol{d}t \sum_{j',k'} c_{ij'}^{k'}\phi_{ij'}^{k'}1_{\{ij'\in G^{k'}\}} \tag{1}$$

There can be many equilibria, or profiles of stakes of cooperation, that satisfy the condition outlined in (1). We focus on the one with the maximal stakes:

Definition 2 (Equilibrium with maximal stakes of cooperation). *An equilibrium with the maximal stakes of cooperation is an equilibrium that achieves the highest possible stakes of cooperation, for all of the agents, among all equilibria. In particular, it is an equilibrium with stakes $\{\phi_{ij}^k\}_{g_{ij}^k=1}$, such that for every other cooperative equilibrium with stakes $\{\widetilde{\phi}_{ij}^k\}_{g_{ij}^k=1}$, we have the following:*

$$\phi_{ij}^k \geq \widetilde{\phi}_{ij}^k,\ \forall i,j,k \ s.t.\ g_{ij}^k = 1.$$

The above stakes of cooperation in such an equilibrium, $\{\phi_{ij}^k\}_{g_{ij}^k=1}$, are called the maximal stakes of cooperations, or MSC's. They solve incentive constraint (1) in equality.

Proposition **1** (Existence of MSC's). *An equilibrium with MSC exists. The set of equilibria forms a complete lattice. Consequently, there exists a largest equilibrium,* the one with MSC's*, and a smallest equilibrium,* the one with the lowest stakes of cooperation, which is 0.

[9]We thank the anonymous referee for suggesting this point.
[10]We discuss the use of grim-trigger strategies at the end of this section.

This proposition follows from the fact that the MSC on all relationships are *complements*: A larger stake of cooperation on any relationship weakly increases the punishment for deviation, thereby benefiting all other relationships.

This proposition illustrates an important observation that the maximal stakes of cooperation for all relationships can be achieved simultaneously. Therefore, it is unambiguous to define "the largest equilibrium" as the best among all equilibria for everyone: the equilibrium with the maximal stakes of cooperation. This will be our focus throughout the paper. We will first characterize the MSC's of a given network structure and then endogenize network formation, exploring the impact of adding a relationship on MSC's.

We also note that the network G can be represented by a weighted network, such that $w_{ij} \equiv \sum_k 1_{\{ij' \in G^{k'}\}}$. The incentive constraint (1) can be reformulated as follows:

$$(\phi_{ij})^2 \leq \int_0^\infty e^{-rt} \lambda \mathbf{d}t \sum_{j'} w_{ij'} \phi_{ij'} \tag{2}$$

A simple algorithm for identifying the largest equilibrium. We provide an algorithm for finding the largest equilibrium here. At each step t of the algorithm, let $\phi_{ij,t}$ be the stakes of cooperation for every relationship on pair ij. Initialize $\phi_{ij,0} = \min\{d_i, d_j\}$, which is an upper bound of the self-enforcing stake. At step $t \geq 1$:

(i) For each ij s.t. $w_{ij} > 0$, define $\phi_{ij,t}$ to be the supremum of ϕ_{ij} such that the incentive constraint (2) holds for ij, given $\phi_{-ij,t-1}$. That is, $\phi_{ij,t}$ solves the following equation:

$$(\phi_{ij,t})^2 = \frac{\lambda}{r} \left[\phi_{ij,t} + \min\{\sum_{j' \neq j} w_{ij'} \phi_{ij',t-1}, \sum_{i' \neq i} w_{i'j} \phi_{i'j,t-1}\} \right]$$

(ii) Terminate if $\phi_{ij,t} \equiv \phi_{ij,t-1}$ for all ij. Otherwise, return to step (i).

When the algorithm terminates, it finds the largest equilibrium.

Monotonicity. We next show a monotonicity result that will be used later.

LEMMA **1** (Monotonicity). *Consider two societies $G_2 \supset G_1$,*[11] *then the "super" network G_2 has (weakly) larger maximal stakes of cooperation:*

$$G_2 \supset G_1 \implies \phi_{ij}^k(G_2) \geq \phi_{ij}^k(G_1), \ \forall i, j, k.$$

While having denser networks always benefits everyone, it is important to investigate which society structures are more effective in supporting cooperation. This is what we do next. We will illustrate the effectiveness of multiplexity and then compare it to standard community enforcement.

[11] That is, $ij \in G_1^k$ whenever $ij \in G_2^k$.

Discussion of Grim-Trigger Strategies. We conclude this part with some discussion.[12] Although simple, and to some extent extreme, grim-trigger strategies provide a useful theoretical tool for our analysis. Critics argue that these strategies are often too harsh for realistic settings Embrey, Fréchette and Stacchetti (2013) or lack renegotiation-proofness Bolton (1990). However, their straightforward nature allows us to focus on comparing multiplexity with linking to a stranger across various complex network structures within our perfect monitoring framework. The introduction of shocks or mistakes might render grim-trigger strategies unsuitable. Nonetheless, as our primary focus is on comparing multiplexity with linking to strangers, this critique applies equally to both scenarios. Ultimately, expanding the analysis to include more lenient forms of punishment, particularly relevant in cases of imperfect monitoring, represents an important avenue for future research. We also direct readers to the discussion at the end of the previous section, where the implications of introducing imperfect monitoring on the tradeoff between multiplexity and community enforcement are explored.

2.2 Multiplexity as Social Collateral

Before endogenizing network formation, we first examine how multiplexity, i.e., having more than one relationship between two agents, affects cooperation incentives. We demonstrate that multiplexity supports larger stakes of cooperation (compared to the single relationship case) because different relationships serve as social collateral for each other. Having an additional relationship enhances the incentives not only in the new relation but also reinforces the incentives in *every* existing one. In this sense, multiple relationships provide incentive spillover and act as complements to each other in boosting incentives.

We begin with the following benchmark example.

EXAMPLE **1** (One pair of agents, single relationship)**.** *Consider a two-agent society with a single relationship. In this case, the maximal stake of cooperation, MSC, solves the following equality:*

$$c(\phi + \phi^2) = c(\phi + \phi \int_0^\infty e^{-rt} \lambda \boldsymbol{d}t)$$

that is,

$$\phi = \frac{\lambda}{r}$$

[12] We thank the anonymous referee for suggesting this discussion.

Figure 2: One pair of agents, single relationship

In this single-pair, single-relationship example, the compatibility index c has no effect on the stakes of cooperation. As we will see later (e.g., in Section 3.1), it may shape agents' incentives when more than one link or relation is involved. Throughout this section, we assume $c = 1$ on all relationships.

Now, we illustrate the forces of multiplexity: having more than one relationship between a pair of agents.

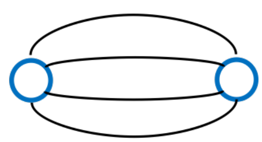
Example **2** (Multiplexity on a single pair)**.** *Consider a two-agent society with not just one, but $k > 1$ relationships. In this case, the maximal stake of cooperation, MSC, solves the following equality:*

$$\phi + \phi^2 = \phi + k\phi\frac{\lambda}{r}$$

that is,

$$\phi = k\frac{\lambda}{r}.$$

Figure 3: Multiplexity between one pair, $k = 4$



We observe that having k relationships increases the maximal stakes of cooperation by a factor of k compared to the scenario with a single relationship. This is because, in multiplexity, k relationships act as social collateral for each other; once an agent deviates in one relationship, they risk losing all k relationships. Moreover, the maximal stake of cooperation (MSC) applies to every individual relationship. Therefore, with k relationships, the maximal stake for each relationship becomes k times larger. As a result, the equilibrium payoff in a scenario with k relationships is k^2 times greater than in the single relationship case. This phenomenon, where the additional relationship not only benefits itself but also enhances all existing relationships, is what we term the *multiplex effect*.

2.3 Multiplexity vs. Community Enforcement

We have illustrated how multiplexity affects cooperation incentives. Another more extensively studied mechanism to enforce cooperation is community enforcement. In the literature, community enforcement refers to situations in which deviations are punished by other members of society (Kandori, 1992). We show that in a complete network, where every agent is connected with everyone else, community enforcement and multiplexity could have the same effect in bolstering incentives.

EXAMPLE **3** (Community enforcement in a complete network). *Consider a complete network with $k+1$ agents, each with one relationship on every link. Each agent has a total degree of $d_i \equiv k$. Since all agents have the same (total) degree, it can be shown that MSCs are the same on all links. The MSC supported by community enforcement solves the following equation:*

$$\phi + \phi^2 = \phi + k\phi\frac{\lambda}{r}$$

that is,

$$\phi = k\frac{\lambda}{r}$$

It follows from the comparison between Examples 2 and 3 that multiplexity provides the same strength of incentives as community enforcement, given the same total degrees.[13] In this sense, multiplexity and community enforcement both enhance cooperation, acting as different types of social collateral.

However, as we will see in the next example, the effectiveness of community enforcement relies more heavily on the rest of the network.

EXAMPLE **4** (Network structure matters more for community enforcement). *Consider a star network with $k+1$ agents, where the central agent has one relationship with each of the remaining k agents, and there is no relationship among the peripheral agents.*

In this case, although the central agent has strong incentives to cooperate, the peripheral agents' incentives are weak and determine the maximal stakes on every relationship. The MSC in this example for every relationship is as follows:

$$\phi + \phi^2 = \phi + \phi\frac{\lambda}{r}$$

that is,

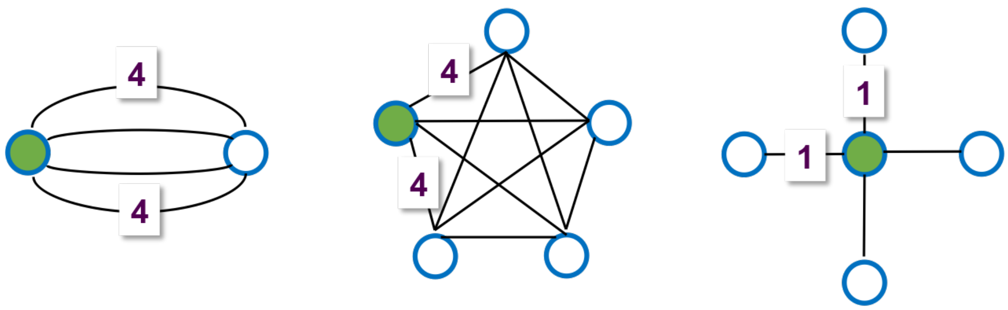
$$\phi = \frac{\lambda}{r}.$$

Thus, the MSC in the star network is equal to that in the single-pair, single-link network, as in Example 1.

[13] Recall that, one's total degree is defined as their total number of links across all relationships.

We conclude this section with the figure below. From left to right, the subfigures depict Examples 2 to 4, respectively. The key messages from this figure are: (1) multiplexity leads to incentive spillover, reinforcing incentives in every existing relationship; and (2) in contrast to community enforcement, multiplexity's effectiveness is less dependent on the overall network structure.

Figure 4: Maximal stakes of cooperation (MSC's) in Examples 2 to 4.



Note: Each subgraph depicts the network structure and the maximal stake of cooperation (MSC) per link (the numbers, on representative links), with $\lambda/r = 1$ and $c = 1$. Multiplexity with 4 relationships and community enforcement with a clique of size 5 (so each agent has a degree of 4) provide the same level of cooperation. Community enforcement relies on the rest of the network structure, as shown in the third (star) network: although the central, green agent has 4 links, the stakes of cooperation are determined by the peripheral agents whose incentives are more binding.

Having explored two key mechanisms for enforcing cooperation—multiplexity and community enforcement—we now pose a critical question: When faced with the opportunity to form a new relationship, which enforcement mechanism should an agent opt for? This dilemma underscores the fundamental tradeoff agents confront in deciding whom to connect with upon the emergence of a new link.

3 Network Formation: Multiplex or Link with a Stranger?

This section introduces a straightforward framework for network formation. We investigate the following question: When presented with the opportunity to establish a new relationship, should an agent link with a friend (i.e., to multiplex) or with a stranger? Additionally, we examine how existing network structures influence this decision.

Consider this network formation process: Starting from a given society G, suppose there is an agent needing to add a new relationship who can propose it to any other agent in the society. For instance, a scholar with a novel research idea may choose to collaborate with an existing co-author or a stranger researcher they have never worked with before.

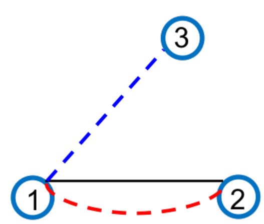
Agents make linking decisions aimed at maximizing their equilibrium payoff in the resultant network structure. In doing so, agents contemplate the implications of adding one new link, but they do not account for potential future links (added either by themselves or others). This assumption is justifiable in contexts with substantial uncertainty about link dynamics or when new relationships are rare.[14]

Finally, let us revisit the monotonicity principle, which posits that having more links always benefits everyone (Lemma 1). In the absence of linking costs, an agent will invariably seek to add a relationship whenever possible, and the recipient will always accept the new link. This premise shifts our focus to questions like "with whom to link" rather than "whether to add a link."[15]

3.1 Multiplexity Trap

In this subsection, we demonstrate that the incentives for multiplexing can be quite substantial. Consider a simple scenario involving three agents, as depicted in Figure 5. Agents 1 and 2 are already in one relationship, while agent 3 has no existing links with anyone in the society. If agent 1 comes across a new opportunity, such as a novel research idea, should she choose to collaborate with her existing partner, agent 2, or opt to work with the unconnected agent 3?

Figure 5: Multiplexity vs. stranger: a three-agent example



Note: Agents 1 and 2 already share one relationship (black line). When agent 1 has a new relationship to establish, will she link with her current friend, agent 2 (red dashed line), or with the stranger, agent 3 (blue dashed line)?

[14]For further justification of this assumption, refer to Jackson (2005). Additionally, our results remain valid even if agents are not completely unaware of future links, provided that new linking opportunities are infrequent and occur over time.

[15]Incorporating linking costs into our model is straightforward. If these costs are uniform across agents, they would not impact the choice between linking with a friend or a stranger.

As it turns out, agent 1 strictly prefers to multiplex with agent 2 rather than link with agent 3. By linking with agent 2, both of their relationships will be enhanced due to the multiplex effect. Specifically, the maximal stake of cooperation between agents 1 and 2, ϕ_{12}, becomes $2\lambda/r$. With two relationships in play, agent 1's total payoff from choosing to multiplex is $4\lambda/r$.

However, if agent 1 links with agent 3, despite having two links, agents 2 and 3 each maintain only one relationship, leading to weaker incentives compared to agent 1's. Consequently, the maximal stakes of cooperation for each link, ϕ_{12} and ϕ_{13}, are determined by the incentive constraints of agents 2 and 3. Since both agents 2 and 3 have only one relationship, $\phi_{12} = \phi_{13} = \lambda/r$, resulting in a total payoff for agent 1 of $2\lambda/r$, which is less than $4\lambda/r$.

Interestingly, this outcome persists even if agents 2 and 3 are also linked. One might assume that agent 1 would be motivated to complete the cycle, i.e., choose to link with agent 3 instead of agent 2 in the presence of a new relationship. However, in such a case, she is indifferent between linking with agent 2 and 3: in either scenario, her payoff is $4\lambda/r$.

Efficient vs. Equilibrium Network Formation In the example above, agent 1 has equal compatibility in linking with both agents 2 and 3. That is, the stage game payoff for cooperative pairs 12 and 13 is identical. In this case, the efficient network coincides with the equilibrium network formation—it is both efficient and an equilibrium choice for agent 1 to link with agent 2. However, in the co-author example, if the new research idea includes an empirical component, and agents 1 and 2 are theorists while agent 3 specializes in empirical analysis, the benefits of cooperation might differ significantly between pairs 12 and 13.

In the following, we demonstrate that when the "compatibility index" c varies across pairs and relationships, the efficient network and the equilibrium network may diverge. We illustrate in the next example that a "multiplex trap" can occur, where agents continue to multiplex even when it would be more efficient to link with strangers.

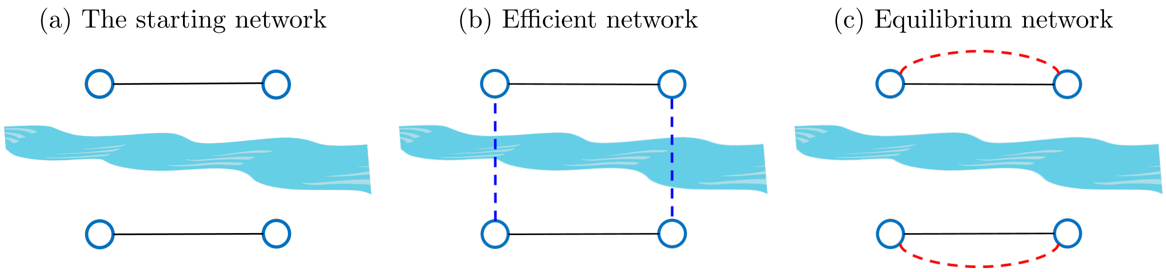
Example **5.** *Consider two villages located on opposite sides of a river, with each village having two villagers. Each villager can engage in two types of relationships: babysitting and trade. It is assumed to be more convenient to babysit within the same village, while trade is more valuable between villages. Let $k = 1, 2$ denote the relationships of babysitting and trade, respectively, with compatibility indices:*

$$c^1_{same} = c^2_{cross} = h \text{ and } c^1_{cross} = c^2_{same} = l, \text{ with } h > l > 0.$$

Suppose the villagers have already formed the babysitting links efficiently (Figure 6a). How will they form the new trade relationship? Will the resulting network be efficient?

While the efficient network would have villagers trading across villages (Figure 6b), villagers in equilibrium prefer to multiplex with friends (Figure 6c), as long as the difference in *compatibilities is not too large, $h/l < 2.25$.*

Figure 6: Efficient vs. Equilibrium Networks.



Note: Blue band represents the river. Solid lines represent relationship 1 (babysitting), which are already formed. Dashed lines represent relationship 2 (trade). The equilibrium network holds for $h/l < 2.25$.

Source of Inefficiency. The source of inefficiency arises from the agents' uncertainty regarding how others will form links. If a villager knew with certainty that a villager in another village would efficiently and immediately form a trade relationship, they would also make the efficient choice. However, due to the inability to foresee future link dynamics—either because the network is too large or new relationships are rare—the multiplex effect tends to dominate the benefits of linking with a stranger, as long as the compatibility gap is not too large.

This observation can be extended to any society, not limited to just four agents, that starts with isolated pairs.

PROPOSITION **2** (Multiplexity Trap)**.** *Starting from a society G_0 consisting of isolated pairs, each with at least one existing relationship rated at h, every agent strictly prefers to multiplex. Consequently, the network remains as isolated pairs, provided the compatibility differences are not too pronounced; in particular, $\frac{\max_j c_{ij}^k}{\min_j c_{ij}^k} < 2.25$, for all i, k.*

Proof. See the Appendix. □

When forming new links, agents face a tradeoff between strong incentive provision through multiplexity and high compatibility. The above proposition underscores that the incentive benefits can be substantial enough for agents to continue multiplexing, even when their current neighbor is not the most suitable for any new relations. This effect is robust: agents prefer to multiplex and ignore the (in)compatibility as long as the compatibility with an existing neighbor is at least about 38% of that with the most compatible strangers.[16]

[16]This is a sufficient condition. It requires that the compatibility of one's current neighbor in G_0 not be

A pair of agents are incentivized to add more relations between themselves and to leverage their existing relations. Such incentives intensify when the pair of agents have multiple relations, leading to what we call a "multiplex trap:" once the process of multiplexing begins, agents have increasing incentives to continue, making it difficult to escape from this pattern.

However, not all societies start from isolated pairs. How do agents choose between multiplexing and linking with a stranger in more complex network structures? We explore this question in the next section. To concentrate on network structures, we revert to uniform compatibility, i.e., $c_{ij}^k \equiv 1$ for all i, j, k.

4 Multiplex or Not: General Network Structures

In this section, we focus on identifying two key network features that influence multiplexity: degree dispersion and assortativity. Degree dispersion refers to the variation in the number of connections (degree) among agents. Assortativity, on the other hand, indicates the tendency of agents to form connections with others who have a similar number of connections.

4.1 Multiplexity Dominates When Degree Dispersion is Low

We start by considering regular networks, where each agent has the same degree, denoted by d. In such networks, there is no degree dispersion.

Lemma **2** (MSC in Regular Networks)**.** *In any society G_0 that is regular with degree d, where every agent's total degree $d_i \equiv d > 0$, the maximal stake of cooperation (MSC) on every relationship is:*

$$\phi(d) = d\frac{\lambda}{r}.$$

In regular networks, agents facing deviation lose the same number of relations, ensuring uniform incentives to cooperate. The MSC becomes $\phi = d\frac{\lambda}{r}$, the total degree multiplied by the MSC supported by a single relation.

Proposition **3** (Multiplexity Dominates in Regular Networks)**.** *In any regular society G_0 with $d_i = d > 0$ for all i, when a new relationship arises, every agent strictly prefers to multiplex.*

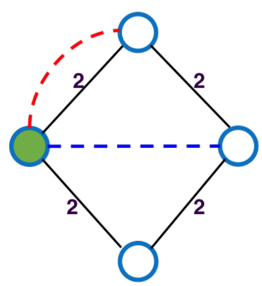
Upon adding a new link ij, agents i and j each achieve a total degree of $d + 1$, supporting larger cooperation stakes between them. If i and j are already linked (i.e., ij is a

significantly lower than that of a potential new link candidate. Furthermore, if one currently maintains more than one link with their current neighbor, the incentives for multiplexity are further strengthened, allowing for a relaxation of this condition.

multiplex link), this increase in incentives benefits *all* their relations, both new and existing. Conversely, if i and j were previously strangers, the enhanced incentive only applies to the new relationship. Importantly, given the uniform degree prior to adding the new link, the cooperation size on the new link between strangers is insufficient to surpass the multiplex effect between friends.

Consider the following regular network with $N = 4$ and $d = 2$ (Figure 7): Before a new relationship arises, each agent has a degree of 2. Per Lemma 2, the MSC on each link is $2\frac{\lambda}{r}$. If the green agent adds a new link, they can choose to multiplex (red dashed line) or link with a stranger (blue dashed line). In either case, the MSC on the new link will exceed $2\frac{\lambda}{r}$. However, if the green agent links with a stranger, the new link's MSC is constrained by the society's incentive short points. In contrast, multiplexing leads to one link at MSC $2\frac{\lambda}{r}$ and another at a higher MSC, thus making the MSC on the multiplex link greater than that on the stranger link.

Figure 7: Multiplexity vs. Stranger in a Regular Network



Note: This regular network has $d = 2$. The MSC for each link is $2\frac{\lambda}{r}$. In the graph, $\frac{\lambda}{r}$ is omitted for simplicity. The green agent has a new link to add, choosing either a current friend (red dashed line) or a stranger (blue dashed line).

4.2 Multiplexity and Assortativity

We have observed that agents tend to multiplex in networks with low degree dispersion. What happens in networks with large degree dispersion? We find that in such networks, the agents' incentives to multiplex depend on the degree to which individuals with similar degrees are connected. In other words, in networks characterized by large degree dispersion, the incentives for multiplexing are influenced by the network's assortativity.

For illustrative purposes, consider networks in which agents have two levels of total degrees. That is, $d_i = d$ or D for all i and $D > d$. In addition, suppose the fraction of agents having degree d is $D/(D + d)$, meaning that it is feasible for agents to only match with others of different degrees. There are two (extreme) cases:

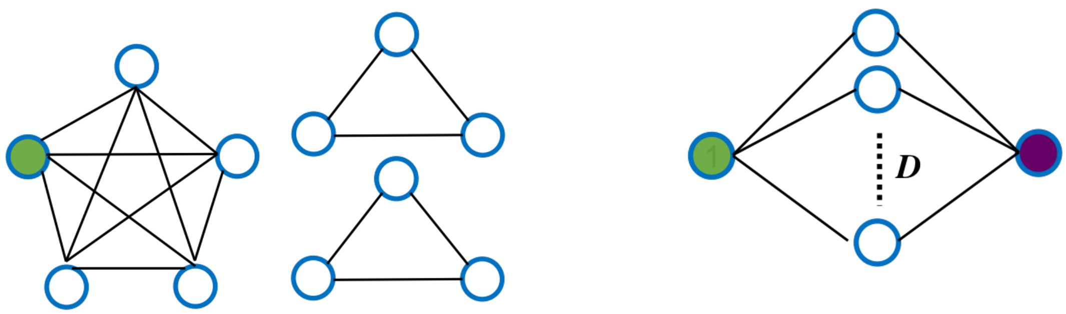
(a) Positively assortative: all links are between agents with the same degree.

(b) Negatively assortative: all links are between agents with different degrees.

An example is depicted in Figure 8.

Figure 8: Networks with different assortativity

(a) Positive assortativity (b) Negative assortativity



Note: This figure plots two networks with different assortativity: positive assortativity in panel 8a and negative assortativity in panel 8b. In both networks, an agent has either a high degree of $D > 2$ or a low degree of $d = 2$. When an agent (colored in green) has a new relationship to add, multiplexity dominates in panel 8a, whereas they prefer to link with a stranger (colored in purple) in panel 8a when D is large enough (≥ 6).

We begin with the positive assortativity case (Figure 8a). We show below that agents strictly prefer to multiplex when networks exhibit positive assortativity.

Proposition 4 (Multiplexity dominates in positive assortative networks). *Starting with any (perfectly) positive assortative society G_0, every agent strictly prefers to multiplex.*

It is important to note that the agents with a low degree d also strictly prefer to multiplex rather than linking with someone with a higher degree. To see this, suppose a degree-d agent establishes a new link with a degree-D agent. Adding this link is inferior to multiplexing for two reasons: (1) the MSC on the new link is limited by the lower-degree agent, whose incentives are weaker, and (2) the MSC's on all existing links do not benefit from the newly added link. This serves as an interesting separating point between our paper and the literature, which usually assumes or predicts that people always prefer to link with agents with a large/higher degree (e.g., see discussions in Jackson et al., 2008 about preferential attachment).

Implication. One implication of the above proposition is that multiplexity and assortativity reinforce each other: multiplexity is very likely to occur in a (positively) assortative network, so more relationships are added between agents with similar degrees; hence, the network becomes more assortative.

However, when networks exhibit negative assortativity, there are situations in which agents are willing to link with a stranger rather than multiplexing, as we see in the next example.

Proposition 5. *Consider the negatively assortative network depicted in Figure 8b. The green agent, who has a current degree of D, only has low-degree ($d = 2$) neighbors. When they have a new relationship to add, they prefer to link to a stranger with degree-D rather than to multiplex when $D \geq 6$.*

In this society that is negatively assortative, all the current neighbors of the high-degree agents have a lower degree ($d = 2$) and hence, can only support relatively small stakes of cooperation. Although multiplexity helps increase the stake of cooperation, a new link with a stranger who has a high degree can be very powerful in supporting a high MSC. When D is large enough ($D \geq 6$ in this case), having one large-stake link is more beneficial than multiplexing.[17]

Summary. In this section, we have explored more complex network structures, finding that two network features significantly influence the incentives for multiplexing: degree dispersion and assortativity. Multiplexity tends to be favored in scenarios where all agents have similar degrees (indicating low degree dispersion) or when agents with comparable degrees predominantly form connections with each other (demonstrating positive assortativity). A critical insight from our analysis is that to escape the "multiplexity trap," it is crucial to have asymmetry in degree distribution among neighbors, and not just in society at large. In Section 6, we present empirical evidence that corroborates these theoretical predictions.

5 Extension: Relationships with Varying Importance

In this part, we explore the case when different relationships vary in their importance. For instance, people might value friendship more than other relationships, such as advice-giving or borrow and lending. Once we allow for asymmetry among relationships, we could explore

[17] It's worth noting that in the above example, it's not essential for the low-degree agents to be common friends with the two high-degree agents. And the incentives of the high-degree agents to link with each other is not driven by completing the triangle. This is because information is complete in our setting, so there is no issue of information transmission.

the following question: Given existing relationships, will people link with current friends on more or less important relationships?

We model this by letting the compatibility index vary across relationships. Specifically, recall that agents' cooperation payoff is $c_{ij}^k \phi_{ij}$. Let c_{ij}^k change with relationship k, which shows that different relationships vary in their importance. When $c^x > c^y$, we say relationship x is more important than relationship y.

We have shown in Proposition 5 that agents prefer to link with a stranger rather than multiplexing when networks exhibit negative assortativity, when the number of friends of the stranger exceeds a certain threshold $\underline{D}$. We show below that the threshold $\underline{D}$ increases in the importance index c.

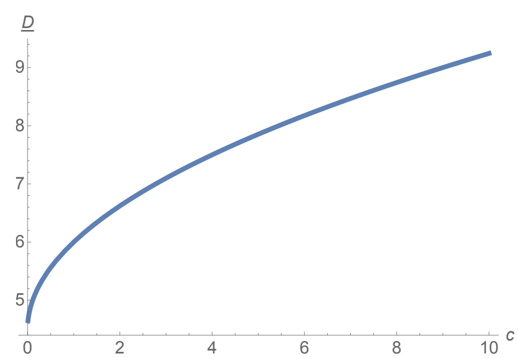
Proposition 6. *Consider a negative assortative network similar to Figure 8b with two levels of degrees: $d \geq 1$ and $D > d$. Suppose a degree-D agent has an opportunity to add a new relationship whose compatibility is $c > 0$, while all the existing relationships/links share the same compatibility of $c_0 = 1$. Let $\underline{D}(d, c)$ be the threshold beyond which linking to a stranger is preferred. Then, we have*

$$\underline{D}(d, c) = \frac{(1+d)^2}{d} + \frac{1+d}{d}\sqrt{c},$$

which strictly increases in c $(\forall d)$. That is, agents are more willing to multiplex when the new relationship is more important.

The above finding shows that the more important the new relationship is, the less willing the green agent (in Figure 8b) will be to link with the purple agent (who is a stranger before the new link is established). In other words, people tend to multiplex with friends on relatively more important relationships. The effect is not negligible because $\underline{D}$ is on the order of $\sqrt{c}$. For instance, when $d = 2$, the threshold $\underline{D}(2, c) = 4.5 + 1.5\sqrt{c}$ and is plotted in Figure 9.

Figure 9: Threshold of adding a link with a stranger increases in its importance.



Note: This figure is for the case of $d = 2$.

Trust and access. The above result sheds light on the tradeoff between trust and access (to information) in the literature (e.g., Coleman, 1988, Burt, 2009, and Karlan et al., 2009). When a new relationship is more important, it requires more social collateral or trust to maintain. Multiplexity provides such social collateral. When the new relationship is primarily for accessing information, i.e., less important and requiring less trust, linking with a stranger becomes more preferable.

6 Empirical Analysis

So far, we have shown that multiplexity could enhance cooperation and that the incentive benefits are so large that we may observe a "multiplexity trap." We further show that agents are more likely to multiplex in networks that exhibit low degree dispersion or positive assortativity. We summarize the above theoretical findings as the following three testable hypotheses:

Testable hypotheses

- Multiplexity prevails in networks (Hypothesis 1).
- Multiplexity is more likely in societies that have:
    - ◇ a low degree dispersion (Hypothesis 2a).
    - ◇ positive assortativity (Hypothesis 2b).

Hypothesis 1 comes from the observation that the multiplexity trap can occur (Proposition 2). Hypotheses 2a and 2b follow from our Propositions 4 and 5.

Data description.

We utilize the Indian Village Survey data from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). These data cover 75 rural villages in Karnataka, an area of southern India within a few hours of Bangalore. The survey contains information about multiple relationships among individuals and households in the same village, which serves as a good starting point to test our theory about multiplexity.

Specifically, we use five types of relationships among the households in our study: (1) going to the temple together; (2) visits during free time; (3) advice giving/receiving; (4) rice/kerosene borrowing/lending; and (5) money borrowing/lending.[18] We treat households

[18] There are 12 relations covered in the original dataset, which are: (1) those who visit the respondent's home, (2) those whose homes the respondent visits, (3) kin in the village, (4) non-relatives with whom the respondent socializes, (5) those from whom the respondent receives medical advice, (6) those from whom the

as nodes in the network.[19] We say a pair of households are connected if at least one member in one household has any of the above relationships with any member in the other household. Accordingly, there are five types of connections. When two households have at least two different types of connections, we say they multiplex with each other.

Testing Hypothesis 1: The Prevalence of Multiplexity.

Our first—and baseline—theoretical prediction is that the multiplexity trap can occur. That is, agents tend to build new relationships on top of existing ones, even when it is not efficient. One implication of this prediction is that multiplexity should be prevalent.

Recall that this prevalence is illustrated in Figure 1, which compares the histogram of the number of other relationships conditional on having a given relationship k or not. For example, Panel 1a shows that, conditional on the household pair ij not having the relationship built on advice, the probability that they do not have any other relationship is 95%. However, this probability drops to 18% when the pair does have the relationship built on advice.

We use the following baseline regression to assess the prevalence of multiplexity:

$$Relation_{ij}^{-k} = \alpha_0 + \alpha_1 Relation_{ij}^{k} + \epsilon_{ij} \tag{3}$$

The key independent variable, $Relation_{ij}^{k}$, is an indicator variable representing whether there is a relationship k between household i and household j in village v. Specifically, $Relation_{ij}^{k} = 1$ if the answer is yes and 0 otherwise. The dependent variable, $Relation_{ij}^{-k}$, indicates whether there is any relationship other than k between the two households, with $Relation_{ij}^{-k} = 1$ if the answer is yes and 0 otherwise. There are five types of relationships, and we let k = temple, visit, advice, money, and rice. Alternatively, $Relation_{ij}^{-k}$ can represent the number of relationships between households i and j other than k, and the results are similar. Table 2 summarizes our results.

We observe that the existence of one relationship between households has a significant and positive impact on whether they have other relationships (Panel A) and on the number of other relationships (Panel B).

respondent would borrow money, (7) those to whom the respondent would lend money, (8) those from whom the respondent would borrow material goods (kerosene, rice, etc.), (9) those to whom the respondent would lend material goods, (10) those from whom the respondent gets advice, (11) those to whom the respondent gives advice, and (12) those whom the respondent goes to pray with (at a temple, church, or mosque). We omitted the kin relation as this is not a choice variable by the agents so that there are 10 relations left, we then regrouped the rest of the relations as we do not distinguish direction of the link, which leaves us with 5 types of relations.

[19] The empirical results are the same if we use individuals as nodes.

Table 1: Prevalence of Multiplexity

	(1)	(2)	(3)	(4)	(5)
Relationship k	Temple	Visit	Advice	Rice	Money
Panel A: Dependent variable: Relationship $-k$ (Yes = 1)					
Relationship k	0.83***	0.71***	0.80***	0.83***	0.81***
(Yes = 1)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Observations	718,951	718,951	718,951	718,951	718,951
R-squared	0.04	0.43	0.30	0.39	0.42
Panel B: Dependent variable is the number of relationship $-k$ (0-4)					
Relationship k	2.60***	1.42***	1.82***	1.74***	1.69***
(Yes = 1)	(0.05)	(0.03)	(0.03)	(0.03)	(0.02)
Observations	718,951	718,951	718,951	718,951	718,951
R-squared	0.067	0.46	0.39	0.46	0.51

Note: The data used are from the Indian Village Survey from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). The OLS results for equation (3) are reported. The regressions are conducted at the household level. The robust standard errors are clustered at the village level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

This provides baseline evidence for *Hypothesis 1*: multiplexity is prevalent in networks. However, there may be various reasons for multiplexity. To further distinguish our theory from previous ones (see Atkisson et al., 2020 for a recent survey), we need to test *Hypotheses 2a* and *2b*. Specifically, we aim to understand how network patterns affect the prevalence of multiplexity.

Testing Hypothesis 2: Determinants of Multiplexity. *Hypotheses 2a* and *2b* concern the determinants of multiplexity. Recall that we have the following theoretical predictions:

- Multiplexity is more likely to occur in societies that have:
  - ◇ low degree dispersion
  - ◇ positive assortativity

To test these hypotheses, we treat each village as an independent observation and follow two steps. First, we measure the level of multiplexity for each village. Then, we use the estimated multiplexity as the dependent variable and the network features (degree dispersion and assortativity) of each village as independent variables.

Specifically, we first conduct the following baseline regression for each village v:

$$Relation_{ij}^{-k} = \alpha_0^v + \alpha_1^v Relation_{ij}^k + \epsilon_{ij}^v \tag{4}$$

This results in a point estimate for α_1^v, $v = 1, \ldots, 75$, for each village. We use this as the measure of the level of multiplexity in village v.

We then provide the measures for the key independent variables of interest, which are as follows. The first variable is degree dispersion. Specifically, for each village, we define the degree dispersion of a network by using the 75th percentile divided by the 25th percentile of the degree among all local households. The higher the value, the more dispersed the network is in terms of degree. The second variable, assortativity, measures how assortative a village is in terms of degree. This variable is defined as the inverse of the average absolute difference between the degrees of connected households in the village. The value of assortativity ranges from 0.1 to 0.3 in the data. A higher value indicates that the villages are more assortative.

To investigate the relationship between multiplexity and network features, we conduct the following regression:

$$Multiplex_v = \beta_0 + \beta_1 D_v + \epsilon_v \tag{5}$$

The dependent variable is the multiplexity measure in village v, which is the estimated α_1^v in equation (4). Let D_v denote the degree dispersion or assortativity measure discussed above. Because we control for village-specific dummies, the main effects of D_v have been absorbed. The coefficient, β_1, is of primary interest because it captures how multiplexity is associated with degree dispersion or assortativity.

Table 2: Prevalence and Determinants of Multiplexity

	(1)	(2)	(3)	(4)	(5)
	Multiplexity				
Measures used	Temple	Visit	Advice	Rice	Money
Panel A: Multiplexity based on all pairs					
Dispersion	-0.178**	-0.0694	-0.135***	-0.0570	-0.117***
	(0.0760)	(0.0532)	(0.0435)	(0.0359)	(0.0353)
Assortativity	0.849**	0.275	0.131	0.383*	0.459**
	(0.423)	(0.295)	(0.235)	(0.206)	(0.184)
Panel B: Multiplexity based on same subcaste pairs					
Dispersion	-0.189*	-0.0788	-0.0566	-0.0450	-0.0287
	(0.0980)	(0.0592)	(0.0396)	(0.0492)	(0.0371)
Assortativity	0.661	0.345	0.0453	0.264	0.107
	(0.569)	(0.303)	(0.258)	(0.252)	(0.208)
Panel C: Multiplexity based on different subcaste pairs					
Dispersion	-0.308***	-0.0621	-0.192***	-0.0628*	-0.162***
	(0.105)	(0.0611)	(0.0607)	(0.0360)	(0.0420)
Assortativity	0.569	0.209	0.209	0.445*	0.570***
	(0.524)	(0.322)	(0.317)	(0.231)	(0.213)

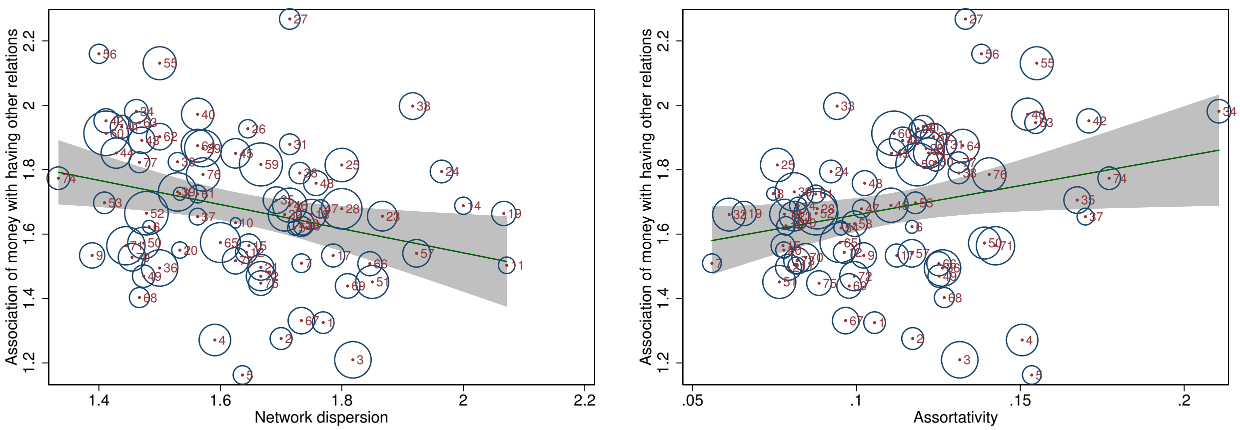
Note: The data used are from the Indian Village Survey from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). All the regressions are conducted at the village level (N = 75). The dependent variable in each column is a measure of multiplexity based on a particular relationship. In each panel, we calculate the multiplexity based on different types of pairs in the sample. The OLS estimates for equation (5) are reported. The robust standard errors are reported in parentheses.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Panel A in Table 2 reports the results. Consistent with our theoretical predictions, we find that degree dispersion is negatively associated with multiplexity, with three out of the five coefficients being statistically significant (*temple*, *advice*, and *money*). The results further show that assortativity is generally positively related to multiplexity at the village level. Figure 10 illustrates this more intuitively.

Figure 10: Determinants of multiplexity, with *money* as the given relationship

(a) Degree dispersion | (b) Assortativity



Note: The data used are from the Indian Village Survey from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). Each point stands for an independent village. The area of the circle indicates the population. For each point, we have the measure for the level of multiplexity (y-axis) and the measure of dispersion (panel a) and assortativity (panel b). The fitted line shows the OLS regression results of equation (5).

Robustness Check

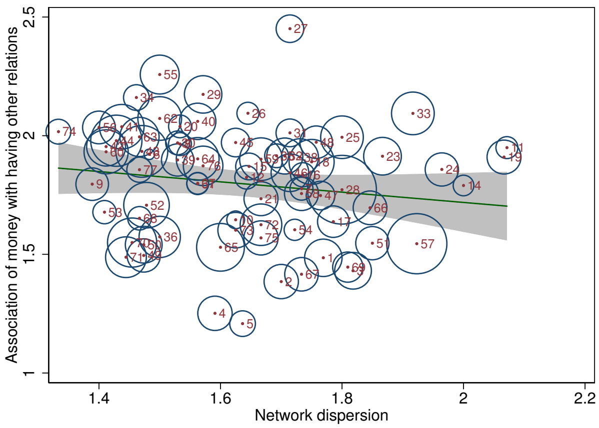
Caste. Given the importance of caste in Indian society, we conduct a parallel investigation for same-subcaste pairs and different-subcaste pairs separately. We conjecture that if cooperation is mostly driven by unmodeled factors among same-subcaste pairs, our theory applies more to different-subcaste pairs because the incentive issue matters more here. This leads to the following robustness check:

(Hypothesis 3) *The effects of degree dispersion and assortativity are stronger (and more significant) for different-subcaste pairs and are weaker (and less significant) for same-subcaste pairs.*

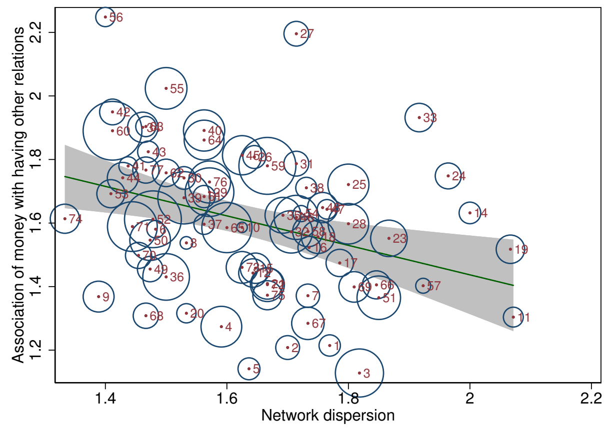
Figure 11 provides evidence supporting *Hypothesis 3*. Panels 11a and 11c are for same-subcaste pairs, and Panels 11b and 11d are for different-subcaste pairs. The associations between multiplexity and degree dispersion/assortativity are more significant among different-subcaste pairs. The regression results are reported in Panels B and C in Table 2, respectively.

Figure 11: Determinants of multiplexity for same-subcaste and different-subcaste pairs

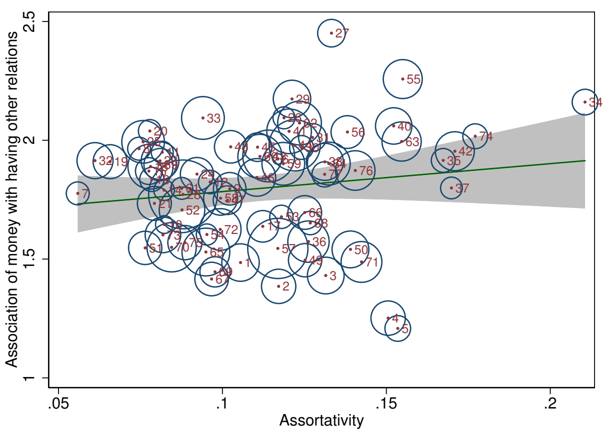
(a) Degree dispersion, same subcaste



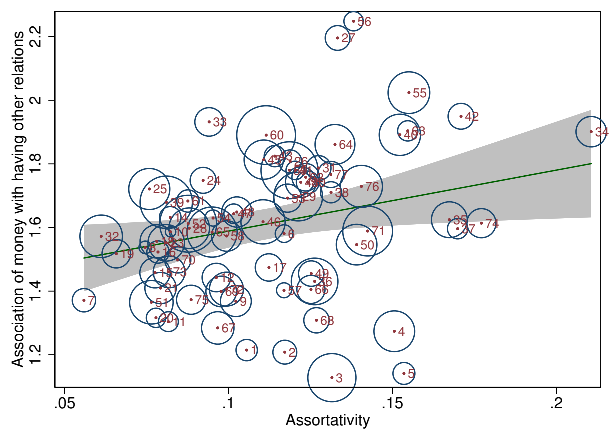
(b) Degree dispersion, different subcaste



(c) Assortativity, same subcaste



(d) Assortativity, different subcaste



Note: The data used are from the Indian Village Survey from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). The figure shows the association of multiplexity with dispersion (Panels a and b) or assortativity (Panels c and d). Different from Figure 10, the measure of multiplexity is only based on the sample with the same subcaste (Panels a and c) or with different subcaste (Panels b and d) in each village.

Homophily. Besides caste, which might be an important factor affecting the pattern of multiplexity, another prominent alternative theory is homophily: agents form overlapping relationships because they share similarities. Using the data available, we approximate the degree of homophily by controlling for whether a pair of agents are from the same occupation type, are both natives in the village, both have a job, have the same level of education, or are of similar ages (within five years). Even after controlling for these measures of homophily, the network features suggested by our theory remain significant in influencing multiplexity. The results are summarized in Table 3.

Table 3: Prevalence and Determinants of Multiplexity with Controls

	(1)	(2)	(3)	(4)	(5)
			Multiplexity		
Measures used	Temple	Visit	Advice	Rice	Money
Panel A: Multiplexity based on all pairs					
Dispersion	-0.242***	-0.128*	-0.151**	-0.0895	-0.124***
	(0.0800)	(0.0707)	(0.0669)	(0.0616)	(0.0536)
Assortativity	1.367***	0.572	0.471	0.652*	0.657**
	(0.496)	(0.399)	(0.383)	(0.366)	(0.313)
Panel B: Multiplexity based on same subcaste pairs					
Dispersion	-0.233***	-0.115**	-0.129***	-0.0433	-0.0481
	(0.0734)	(0.0526)	(0.0450)	(0.0404)	(0.0364)
Assortativity	0.702	0.370	0.398	0.127	0.151
	(0.568)	(0.294)	(0.269)	(0.236)	(0.218)
Panel C: Multiplexity based on different subcaste pairs					
Dispersion	-0.297***	-0.157*	-0.166**	-0.0896	-0.163**
	(0.104)	(0.0886)	(0.0867)	(0.0815)	(0.0646)
Assortativity	1.032	0.978**	0.548	0.800*	1.108***
	(0.622)	(0.458)	(0.436)	(0.464)	(0.358)
Controls	Yes	Yes	Yes	Yes	Yes

Note: The data used are from the Indian Village Survey from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). All the regressions are conducted at the village level (N = 75). The dependent variable in each column is a measure of multiplexity based on a particular relationship. In each panel, we calculate the multiplexity based on different types of pairs in the sample. The OLS estimates for equation (5) with added controls are reported. The controls are measures for 'homophily' and include information regarding whether a pair of agents are from the same type of occupation; are both native in the village; both have a job; have the same level of education; of similar ages (within five years).The robust standard errors are reported in parentheses.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

7 Conclusion

In this research, we delve into the strategic underpinnings of interdependence in multiple relationships. Our focus centers on identifying network characteristics that influence the intersection of various relationships. A pivotal trade-off highlighted in our study is between multiplexity (forming connections within one's circle of friends) and community enforcement. Our analysis yields new hypotheses: networks characterized by low degree dispersion or positive assortativity are likely to favor multiplexity. Furthermore, our findings suggest that individuals are inclined to form connections within their circle of friends, especially when these relationships hold greater significance, resonating with the trust-access dichotomy discussed in existing literature (e.g., Granovetter, 1973, Karlan et al., 2009). Utilizing data from the Indian Village Survey (Banerjee et al., 2013, Jackson, Rodriguez-Barraquer and Tan, 2012), our theories find empirical support.

Our study contributes to the expansive body of literature on the informal economy, primarily focusing on community enforcement, by exploring an alternative mechanism for fostering cooperation: multiplexity. This work adds to the discourse by examining the selection between diverse informal institutions. We provide initial insights into why some societies predominantly nurture relationships within their circle of friends, while others extend cooperation to strangers. While our emphasis is on network characteristics, we acknowledge the significance of other influential factors such as formal institutions, homophily, limited attention, enhanced insurance options, and asymmetric information. Integrating any of these elements into our current analysis presents promising avenues for future research.

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Appendix: Proofs

Proof of Proposition 1. The statement holds by a standard application of Tarski's fixed point theorem, as the stakes of cooperations ϕ's are complements. ∎

Proof of Lemma 1.

Since $G_2 \supset G_1$, incentive constraints (1) for all i, j, k are more restrictive under G_1. Then $\vec{\phi}(G_2)$ would violate some incentive constraints under G_1. Actually, one can identify $\vec{\phi}(G_1)$ by running the algorithm in Section 2.1 (for G_1) and Initializing $\phi_{ij,0} = \phi_{ij}(G_2)$. ∎

Proof of Proposition 2.

To simplify notations, we normalize $\frac{\lambda}{r} = 1$ unless noted otherwise. This normalization does not affect the agent's decision of adding new links, as the same $\frac{\lambda}{r}$ applies to all relationships/links and only add a constant multiplier to every MSC ϕ.

To calculate the threshold, if sufficies to have two levels of compatibilities, $c_H > c_L = 1$. Consider an example of $n = 4$ agents, such that $G^1 = \{12, 34\}$, and $G^2 = \emptyset$. Suppose agent 1 is to add a link in G^2, either 12 or 13. The tendency of multiplexing is minimized when $c^1_{12} = c^1_{34} = c_L$, $c^2_{12} = c_L$, and $c^2_{13} = c_H$; that is, both existing relationships are of low compatibility, and the compatibility of the potential new relationship is high if 1 multiplex, and low if 1 links to the stranger (agent 3).

If 1 adds link 12: $\phi^1_{34} = 1$, and $\phi^1_{12} = \phi^2_{12} = 2$. So

$$\pi_1(G + 12) = \phi^1_{12} + \phi^2_{12} = 4.$$

If 1 adds link 13: $\phi^1_{12} = \phi^1_{34} = 1$ (determined by the incentives of agents 2 and 4), and $\phi^2_{13} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{c_H}}$, which solves $c_H\phi^2 = c_H\phi + 1$. So

$$\pi_1(G + 13) = \phi^1_{12} + c_H\phi^2_{12} = 1 + c_H(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{c_H}}),$$

which increases in c_H.

Let $\overline{c_H}$ be the threshold such that $\pi_1(G + 12) = \pi_1(G + 13)$; that is, $\overline{c_H}$ solves $4 = 1 + c_H(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{c_H}})$. Then $\overline{c_H} = 9/4 = 2.25$.

Therefore, agent 1 strictly prefers to multiplex (on 12) if and only if $c_H < \bar{c} = 2.25$.

To complete the proof, we observe that having more relationships on isolated pairs only make everyone (weakly) more willing to multiplex. In particular, suppose there are $k > 1$ relationships (with low compatibility $c_L = 1$) to begin with on both pairs 12 and 34. Using a similar argument as in above, adding link 13 leads to a stake of $\phi_{13} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k}{c_H}}$, and the threshold $\overline{c_H}$ at which agent 1 is indifferent between adding 12 and 13 solves $2k+1 = c_H(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k}{c_H}})$. Then $\overline{c_H} = \frac{(2k+1)^2}{3k+1}$. Easy to see $\overline{c_H}$ increases in k. ∎

Proof of Lemma 2. Recall the incentive constraint (2) is

$$(\phi_{ij})^2 \leq \frac{\lambda}{r} \sum_{j'} w_{ij'} \phi_{ij'},$$

in which the weights $\sum_{j'} w_{ij'} \equiv d_i = d, \forall i$ by assumption. Therefore, the largest equilibrium features the same stake on every link $\phi_{ij} \equiv \phi(d)$, which solves $\phi(d)^2 \leq \frac{\lambda}{r} d\phi(d)$. Then $\phi(d) = d$. ∎

Proof of Proposition 3. To simplify notations we again normalize $\frac{\lambda}{r} = 1$.

Consider agent 1's decision of adding a new relationship to an agent 2. Suppose $w_{12}(G_0) = k$. In particular, 2 is a neighbor of 1 in G_0 if $k > 0$, and a stranger if $k = 0$.

By assumption, after the newly added link agent 1 and either 2 will both have a degree of $d+1$, and all the other agents still have a degree of d. Therefore, agents other than 1 and 2 have binding incentive constraints on all links/relationships, whose equilibrium stakes remain the same as $\phi_{ij} = d, (\forall ij \neq 12)$.

Now we calculate the stake on link 12. $w_{12}(G_0 + 12) = k + 1$. Then all relationships on pair 12 will have a stake of $\phi_1 2$ which solves

$$(\phi_{12})^2 = (k+1)\phi_{12} + (d-k) \times d.$$

Easy to see $\phi_{12} > d$, and it increases in k: intuitively, there are more relationships between the two high-degree agents (1 and 2) and these high-powered relationships support each other. The more of them, larger the stake on each.

In addition, it follows from the fact $\phi_{12} > d$ that an increase in k also directly benefits agent 1 by having more high-stake links – this is reflected in her payoff function

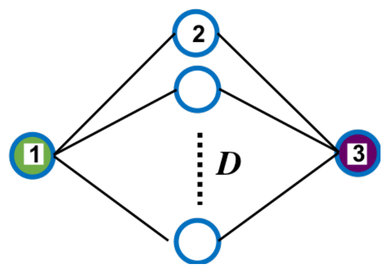
$$\pi_1(G_0 + 12) = (k+1)\phi_{12} + (d-k) \times d.$$

In sum, $\pi_1(G_0 + 12)$ increases in k, the number of existing relationships on pair 12. The same calculation applies to adding any other link. Therefore, agent 1's best choice is to add a link to a neighbor that has most links with her in G_0. ∎

Proof of Proposition 4. This proposition is one step further from Proposition 3: an extremely positive assortative network G can be viewed as having two separate components G_d and G_D, in which all agents in each subcomponent have the same degree d and D, respectively. Proposition 3 suggests that every agent prefers to multiplex than to link with a stranger within the same (degree) group. The proposition further states that agents also prefer to multiplex than to link with someone in the different group. ∎

Proof of Proposition 5. To simplify notations we again normalize $\frac{\lambda}{r} = 1$.

Denote the current society G_0. Let the green agent be agent 1, one of her neighbors (white) be agent 2, and the purple agent be agent 3. We compare 1's payoff between adding a relationship to link 12 (to multiplex) and adding link 13.



Adding 12: The network becomes $G_0 + 12$, in which $d_2 = w_{12} + w_{13} = 2 + 1 = 3$, and agents $d_1, d_3 \geq 3$. Therefore, agent 2's incentives constraint determines the stakes on pairs 12 and 13. Then $\phi_{12} = \phi_{13} = 3$. All other links have a stake of 2 in the society. Thus agent 1's payoff

$$\pi_1(G_0 + 12) = 2\phi_{12} + 2(D - 1) = 2D + 4.$$

Adding 13: The network becomes $G_0 + 13$. All links other than 13 have a stake of 2 in the society. The stake on the new link 13, ϕ_{13}, solves the incentive constraint $\phi^2 = \frac{\lambda}{r}[\phi + 2D]$. So $\phi_{13} = 0.5 + 0.5\sqrt{1 + 8D}$. Thus agent 1's payoff

$$\pi_1(G_0 + 13) = \phi_{13} + 2D = 0.5 + 0.5\sqrt{1 + 8D} + 2D.$$

The threshold $\underline{D}$ that makes 1 indifferent between 12 and 13 solves $2D + 4 = 0.5 + 0.5\sqrt{1 + 8D} + 2D$, hence $\underline{D} = 6$.

In sum, agent 1 prefers to link with a stranger (adding 13) if $D \geq \underline{D} = 6$. ∎

Proof of Proposition 6. To simplify notations we again normalize $\frac{\lambda}{r} = 1$.

We repeat the proof of Proposition 5 by introducing the additional parameter c. The existing relationships in soceity G_0 all have a compatibility normalized to $c_0 = 1$. Again, let the green agent be agent 1, one of her neighbors (white) be agent 2, and the purple agent be agent 3. We compare 1's payoff between adding a relationship to link 12 (to multiplex) and adding link 13.

Adding 12: The network becomes G_0+12. Agent 2's incentives constraint determines the stakes on pairs 12 and 13. Let ϕ_0 be the stake in each of d relationships with compatibility $c_0 = 1$ (those in G_0), and ϕ_c be the stake in the new relationship with compatibility c. ϕ_0 and ϕ_c solve the following equations:

$$(\phi_0)^2 = d\phi_0 + c\phi_c; \quad \text{and} \quad c(\phi_c)^2 = d\phi_0 + c\phi_c.$$

Easy to see $\phi_0 = \sqrt{c}\phi_c$, so $\phi_0 = d + \sqrt{c}$ and $\phi_c = \frac{d}{\sqrt{c}} + 1$. All other links have a stake of d in the society. Thus agent 1's payoff

$$\pi_1(G_0 + 12) = c\phi_c + \phi_0 + d(D - 1) = dD + c + (d + 1)\sqrt{c}.$$

Adding 13: The network becomes $G_0 + 13$. All links other than 13 have a stake of 2 in the society. The new link 13 has a compatibility of c, and its stake ϕ_{13} solves the incentive constraint

$$c\phi^2 = c\phi + dD.$$

So $\phi_{13} = 0.5 + 0.5\sqrt{1 + 4dD/c}$. Thus agent 1's payoff

$$\pi_1(G_0 + 13) = \phi_{13} + dD = 0.5c + 0.5\sqrt{c^2 + 4dDc} + dD.$$

The threshold $\underline{D}$ solves $\pi_1(G_0 + 12) = \pi_1(G_0 + 13)$, hence

$$\underline{D}(d, c) = \frac{(1 + d)^2}{d} + \frac{1 + d}{d}\sqrt{c},$$

which strictly increases in c. ∎